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Linear and Nonlinear Analysis of Fluid Slosh Dampers

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A vibrating structure and a container partially filled with fluid are considered coupled in a free vibration mode. To simplify the mathematical analysis, a pendulum model to duplicate the fluid motion and a mass-spring dashpot representing the vibrating structure are used. The equations of motion are derived by Lagrange's energy approach and expressed in parametric form. For a wide range of parametric values the logarithmic decrements of the main system are calculated from theoretical and experimental response curves in the linear analysis. However, for the nonlinear analysis the theoretical and experimental response curves of the main system are compared. Theoretical predictions are justified by experimental observations with excellent agreement. It is concluded finally that for a proper selection of design parameters, containers partially filled with viscous fluids serve as good vibration dampers.

Nomenclature

C_m = coefficient of the structural damping of the main system
C_p = coefficient of viscous damping of the fluid (pendulum)
D = diameter of the fluid container
G = acceleration of gravity
H = fluid height in the container
K_m = equivalent spring stiffness of the main system
L_p = pendulum arm length
L_s = length of the supporting wires
M = total mass $(M_0 + M_p + M_s + M_t + M_w)$
M_0 = stationary mass of the fluid
M_p = mass of the pendulum (moving mass of the fluid)
M_s = mass of the support
M_t = mass of the fluid container
M_w = mass of the added weight
W_p = weight of M_p
x = response of the main system
β = coefficient of the cubic spring
γ_s = coordinate of the supporting wires
δ = logarithmic decrement
ϵ = x_0/L_p
ζ_p = viscous damping factor
ζ_{pc} = critical value of ζ_p at which the main system exhibits maximum damping
ζ_m = structural damping factor
θ = planar coordinate of the pendulum
μ = mass ratio
ξ = β/W_p
τ = dimensionless time
ω_p = natural frequency of the pendulum
ω_m = natural frequency of the main system
Ω = ω_p/ω_m tuning frequency ratio
Ω_p = frequency ratio Ω at which the damping is maximum, referred to as critical tuning frequency

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Introduction

WHEN a rigid container partially filled with fluid is attached to a vibrating structure, the fluid imparts forces to its container which in turn are transmitted to the vibrating structure influencing its motion. The imparted forces are inertial, gravitational, and dissipative in nature. Use can be made of the inertial forces for vibration absorption and of the dissipative forces for damping. However, the gravity forces add to the weight penalty and, therefore, should be kept at a minimum. It is for the same reason that containers partially filled with viscous fluids are tried as damped vibration absorbers.

Abramson[1] has published the results of many investigators' research work on the subject of fluid motion in rigid containers. Such information is of vital importance for the pursuance of this project. At the same time, linear mechanical modeling of fluid motion in spherical containers as reported by Sumner[2,3] will be used directly in modeling the complex fluid motion. Relevant information on damping of fluids oscillating in spherical containers can be obtained from Stephen's[4] and Sumner's[5] reports.

The coupled dynamic response of containers partially filled with fluids have been studied earlier by Stephen[6] but not with the intention of making vibration absorbers out of oscillating fluids. Chen's thesis[7] is one of the earliest literature in which the cylindrical fluid filled containers are proposed as vibration dampers. Chen's analysis deals with forced vibrations for a more limited range of parameters and design criteria than presented in this paper. However, Bauer[8] has recently applied this method to damp the vibration of satellite antennas by employing cylindrical containers filled completely with two immiscible liquids.

To duplicate the nonlinear response of fluid oscillations, a cubic spring is added to the linear pendulum model[9,10] and the nonlinear equations of motion are obtained by Lagrange's formulation. For the linear analysis only the logarithmic decrements were calculated, while for the nonlinear analysis the Runge-Kutta numerical method was used and a digital computer was employed to calculate the response of the main system. Since the concept of logarithmic decrements becomes meaningless in the nonlinear range, the calculated and experimental response curves of the main system were compared for selected values of parameters.

Mathematical Formulation

Referring to Fig. 1, writing the energy expressions, using Lagrange's energy equation and simplifying, the following equations of motion for the freely vibrating coupled system will be obtained:

$$\ddot{x}+\left(\frac{M_p}{M}\right)L_p\ddot{\theta}\cos\theta+\left(\frac{C_m}{M}\right)\dot{x}+\left(\frac{K_m}{M}+\frac{G}{L_s}\right)x - \left(\frac{M_p}{M}\right)L_p\dot{\theta}^2\sin\theta=0 \qquad (1)$$

$$\ddot{\theta}+\left(\frac{1}{L_p}\right)\ddot{x}\cos\theta+\left(\frac{C_p}{M_p}\right)\dot{\theta}+\left(\frac{G}{L_p}\right)\sin\theta+\left(\frac{G}{L_p}\right)\left(\frac{\beta}{W_p}\right)\theta^3=0 \qquad (2)$$

where $L_s\sin\gamma_s \simeq L_s\gamma_s \simeq x$. Equations (1) and (2), if written in terms of dimensionless parameters, would be of more practical value and, therefore, the analysis will be more general in nature. Letting $X=x/x_0$, $\theta=\theta$, and $\tau=\omega_m t$, substituting into Eqs. (1) and (2), and simplifying, the equations of motion will read:

$$X''+\left(\frac{M_p/M}{x_0/L_p}\right)\theta''\cos\theta+\left(\frac{C_m/M}{\omega_m}\right)X' + \left[\frac{(K_m/M)+(G/L_s)}{\omega_m^2}\right]X-\left(\frac{M_p/M}{x_0/L_p}\right)\theta'^2\sin\theta=0 \qquad (3)$$

$$\theta''+(x_0/L_p)X''\cos\theta+\left(\frac{C_p}{M_p}\right)\left(\frac{1}{\omega_m}\right)\theta'+\left(\frac{G/L_p}{\omega_m^2}\right)\sin\theta + \left[\frac{(G/L_p)(\beta/W_p)}{\omega_m^2}\right]\theta^3=0 \qquad (4)$$

where

$$X'=\frac{dX}{d\tau},\qquad X''=\frac{d^2X}{d\tau^2},\qquad \theta'=\frac{d\theta}{d\tau},\qquad \theta''=\frac{d^2\theta}{d\tau^2}$$

After making appropriate substitutions, the equations of motion will take the following form:

$$X''+\left(\frac{\mu}{\epsilon}\right)\theta''\cos\theta+2\zeta_m X'+X-\left(\frac{\mu}{\epsilon}\right)\theta'^2\sin\theta=0 \qquad (5)$$

$$\theta''+\epsilon X''\cos\theta+2\zeta_p\Omega\theta'+\Omega^2\sin\theta+\Omega^2\xi\theta^3=0 \qquad (6)$$

Linear Analysis

For the linear analysis one can assume

$$\sin\theta\simeq\theta,\quad \cos\theta\simeq 1,\quad \theta^3\simeq\text{small},\quad \theta'^2\sin\theta\simeq\text{small},$$

and after substitutions, Eqs. (5) and (6) can be written as:

$$X''+\left(\frac{\mu}{\epsilon}\right)\theta''+2\zeta_m X'+X=0 \qquad (7)$$

$$\theta''+\epsilon X''+2\zeta_p\Omega\theta'+\Omega^2\theta=0 \qquad (8)$$

Assuming solutions

$$X=Ae^{S\tau}\quad\text{and}\quad\theta=Be^{S\tau}$$

and substituting in Eqs. (7) and (8), the characteristic equation will be obtained.

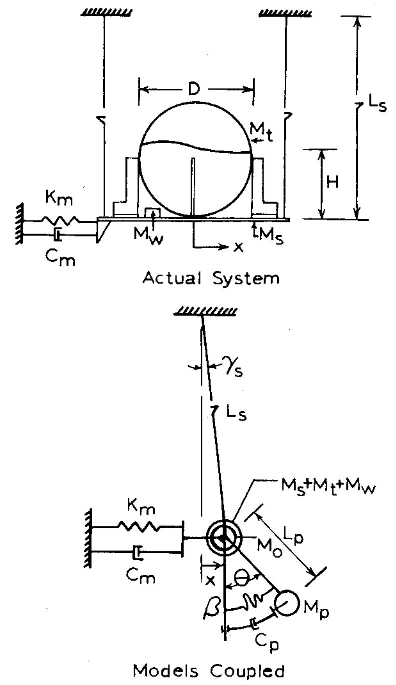

Fig. 1 Two degree of freedom system coupled.

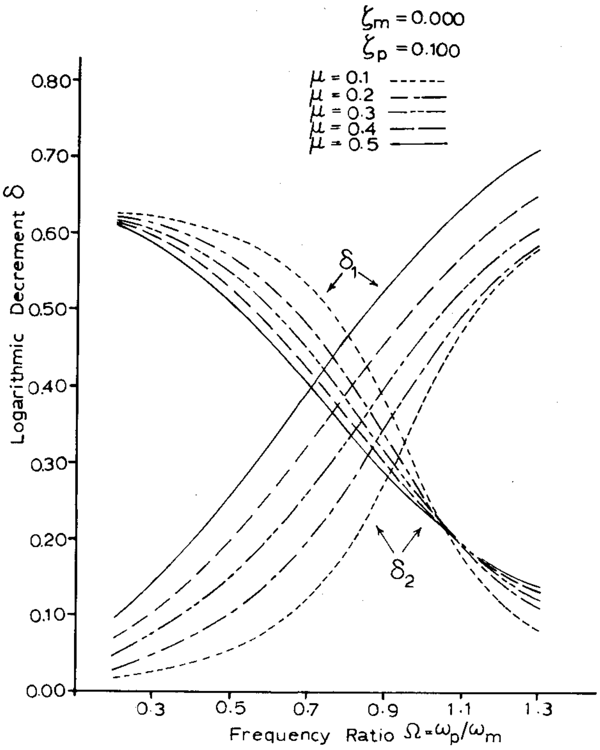

Fig. 2 Logarithmic decrement of motion, $\zeta_p = 0.10$.

$$(1-\mu)S^4+2(\zeta_p\Omega+\zeta_m)S^3+(\Omega^2+4\zeta_m\zeta_p\Omega+1)S^2 +2(\zeta_m\Omega^2+\zeta_p\Omega)S+\Omega^2=0 \qquad (9)$$

The roots of Eq. (9) are generally pairs of complex conjugates and can be written as

$$S_{1,2}=-a_1\pm ib_1,\qquad S_{3,4}=-a_2\pm ib_2$$

After substitution of the above roots into the assumed solutions, the following results:

$$X = e^{-a_1\tau}[C\cos b_1\tau + D\sin b_1\tau] + e^{-a_2\tau}[E\cos b_2\tau + F\sin b_2\tau] \tag{10}$$

$$\theta = e^{-a_1\tau}[H\cos b_1\tau + I\sin b_1\tau] + e^{-a_2\tau}[J\cos b_2\tau + K\sin b_2\tau] \tag{11}$$

Considering the response of the main system only, one can write Eq. (10) as

$$X = C' e^{-a_1\tau}\sin(b_1\tau + \phi_1) + E' e^{-a_2\tau}\sin(b_2\tau + \phi_2) \tag{12}$$

Equation (12) indicates that the response of the main system consists of two decaying components and that for each component there exists a logarithmic decrement $\delta = 2\pi(a/b)$.

To understand which component or, if both, what combination of components of the preceding solution governs the behavior of the main system, a computer program was run to find different roots of Eq. (9). Knowing the roots of Eq. (9) for carefully selected values of design parameters, a variety of curves for δ_1 and δ_2 were plotted. A typical plot of this type is shown in Fig. 2. Based on the preceding analysis Figs. 3-5 were produced. These curves indicate an increase in the damping of the main system for increasing the viscous damping.

Nonlinear Analysis

Quite often the fluid oscillations reach high amplitudes even if the vibration of the main system remains in the linear range. Therefore, the linear approximations assumed earlier will not be valid anymore. Since the validity of a pendulum model for duplication of large amplitudes of fluid motion is justified,[10] the same model will be used for large amplitudes of the coupled motion.

The nonlinear Eqs. (5) and (6) were written in the following form:

$$X'' = f(X', X, \theta', \theta, \tau) \tag{13}$$

$$\theta'' = g(\theta', \theta, X', X, \tau) \tag{14}$$

A digital computer and the Runge-Kutta numerical method were used to solve the given nonlinear equations of motion. Typical response curves are presented in Figs. 6 and 7 which also reveal improvements in the damping of the main system due to increased viscous damping.

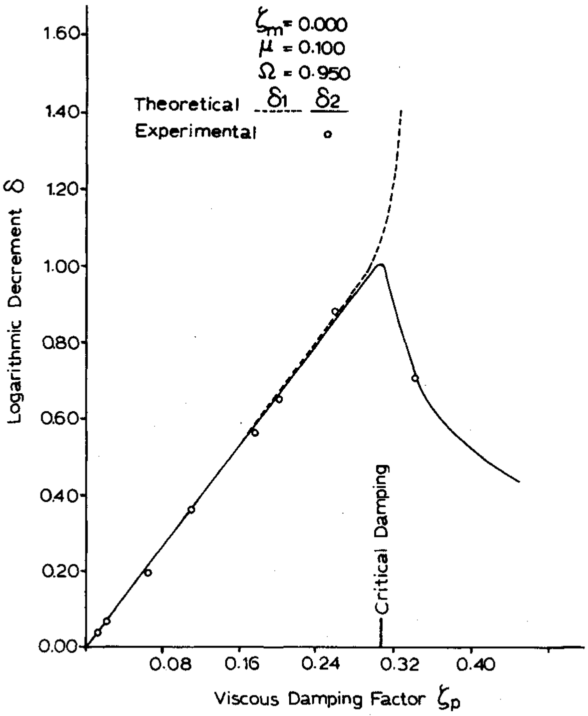

Fig. 4 Influence of viscous damping on the damping of the main system, $\Omega = 0.95$.

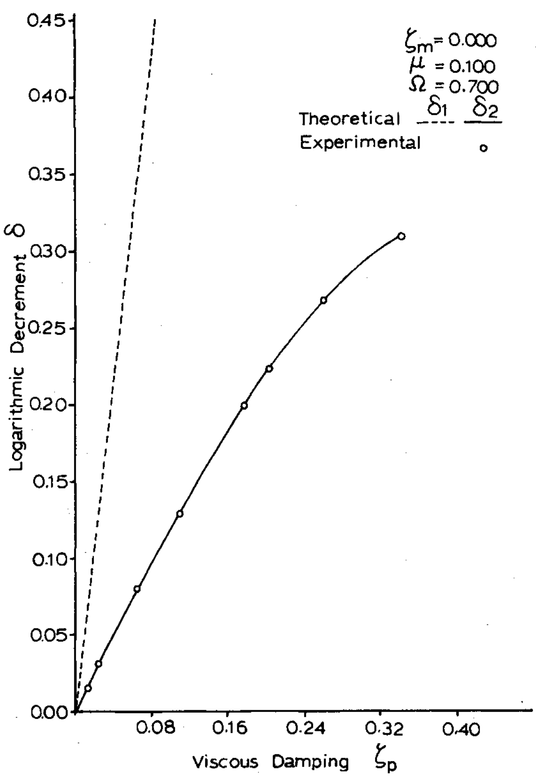

Fig. 3 Influence of viscous damping on the damping of the main system, $\Omega = 0.70$.

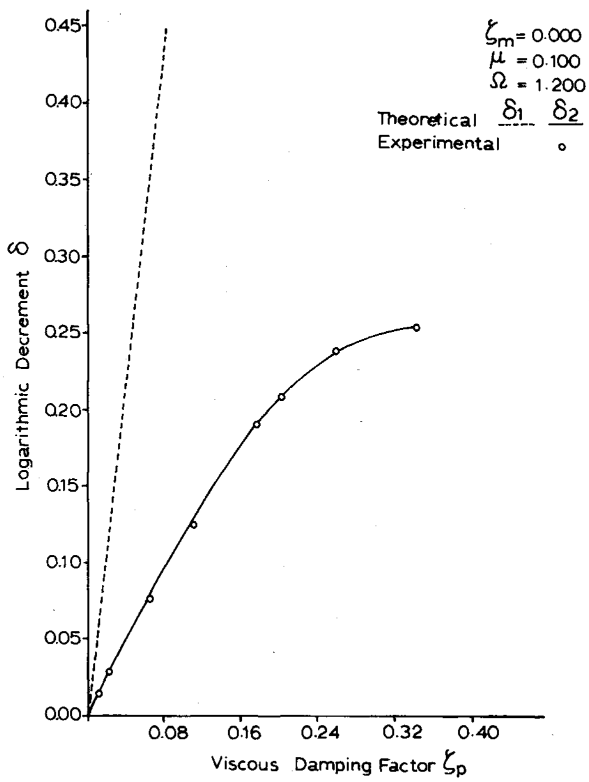

Fig. 5 Influence of viscous damping on the damping of the main system, $\Omega = 1.20$.

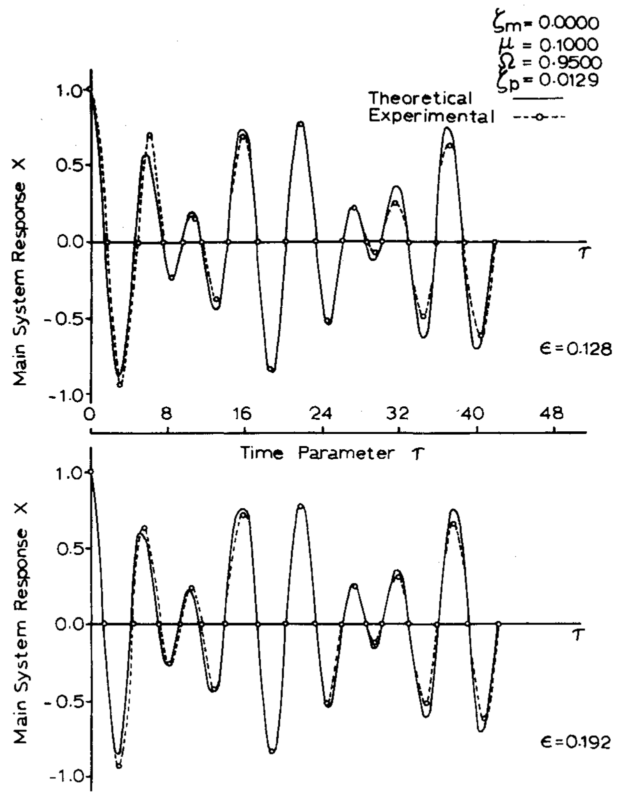

Fig. 6 Theoretical and experimental response of the main system for large initial displacements, $\zeta_p = 0.0129$.

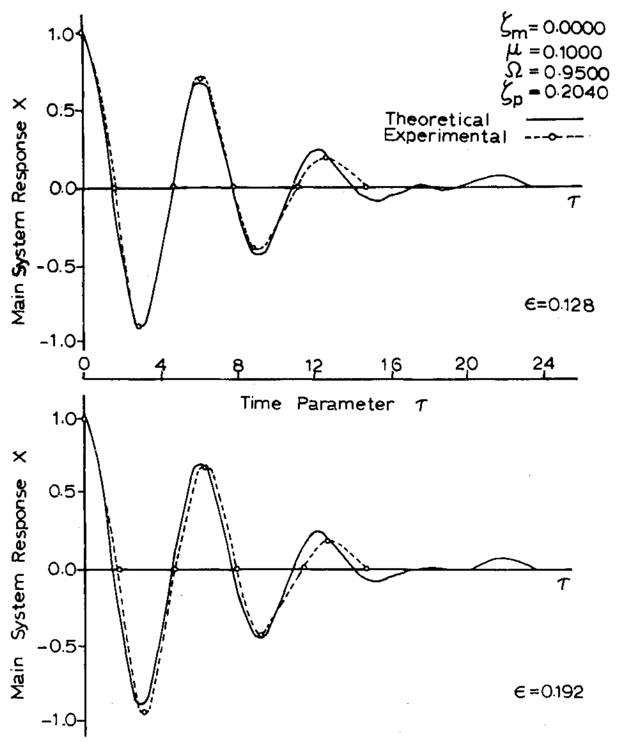

Fig. 7 Theoretical and experimental response of the main system for large displacements, $\zeta_p = 0.2040$.

Experimental Investigations

An experimental setup as shown in Fig. 8 was constructed and the response of the main system for various parametric values were recorded. The recordings were produced by an x-y plotter connected to the photo cell displacement transducer[11,12] and occasionally by a Polaroid Land camera attached to the oscilloscope. Parameters were varied by changing the fluid height and fluid viscosity of the auxiliary system and the initial displacements of the main system. The frequencies were tuned by changing the springs of the main system, and adjustments in the mass ratios were made by adding weights with due consideration of their effects on the frequencies of the main system.

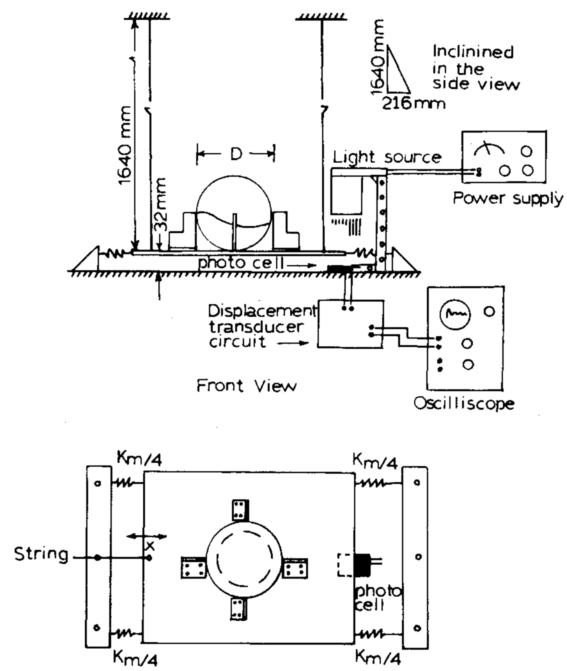

Fig. 8 Experimental setup for free vibration.

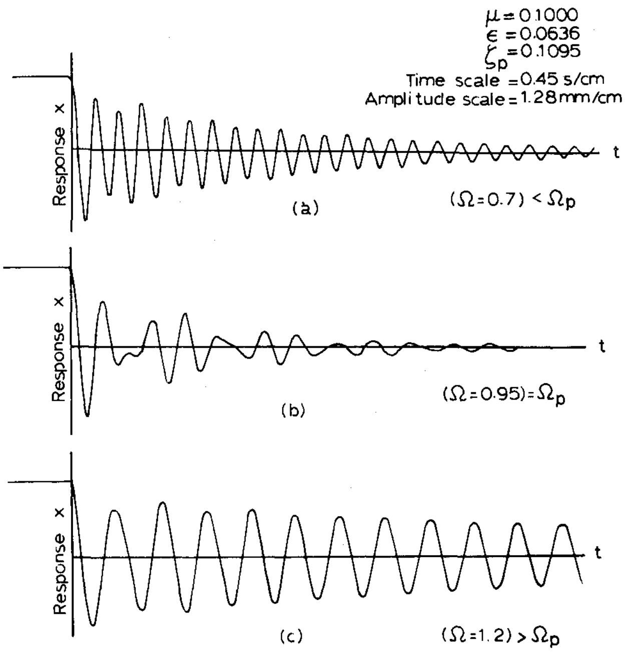

Fig. 9 Response of the main system at and on either side of the critical tuning frequency.

For the linear analysis, the logarithmic decrements of the main system were calculated from the measured response and plotted against the theoretical curves of Figs. 3-5. The excellent agreement between the experimental results and mathematical predictions certify the correct formulation of the problem.

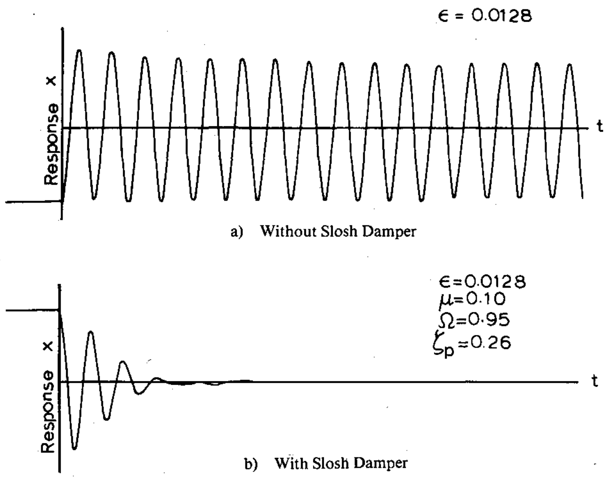

Fig. 10 Response of the main system before and after the application of slosh damper.

Selected experimental response curves as presented in Figs. 9 and 10 indicate the existence of "beat" at the vicinities of tuning frequency, and significant damping obtained by application of fluid dampers as compared to the response of the main system without the application of slosh dampers.

The concept of logarithmic decrements becomes meaningless in the nonlinear analysis; therefore, the experimental response curves were compared to theoretical curves as shown in Figs. 6 and 7. The agreement, being acceptable, reveals the fact that the damping of the main system increases with increased damping of the auxiliary system.

Conclusions

From the mathematical and experimental analysis presented the following conclusions can be drawn:

1) For frequency ratios below the critical tuning and those above the critical tuning, one component of Eq. (12) vanishes rapidly due to high damping while the other component describes the motion of the main system.

2) For frequency ratios at the critical tuning, both components of Eq. (12) have the same logarithmic decrement in the linear range and the main system exhibits maximum damping.

3) In the immediate vicinities of the critical tuning frequencies the "beat" phenomenon occurs.

4) At the critical tuning frequency, the damping of the main system is linearly proportional to the damping of the auxiliary system up to a critical value after which it decreases with further increase in the fluid damping.

5) In the vicinities of tuning frequencies, the damping of the main system increased nonlinearly with the damping of the auxiliary system both for small and large amplitudes of vibration.

6) It was observed that the nonlinearities were mainly due to the large fluid amplitudes and not from the vibration of the main system.

7) The analysis covered a range of accelerations up to 75% of gravitational acceleration G. It is predicted that for accelerations greater than G, the fluid will particulate and the proposed nonlinear model would not describe accurately the behavior of the fluid motion. However, experimental evidences confirm the additional damping obtained due to fluid's breaking waves at high G. For theoretical analysis, a fluid impact modeling, which could be made the subject of another investigation, is suggested.

8) For appropriate selection of parameters, containers partially filled with viscous fluids can be employed as damped vibration absorbers.

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